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ABSTRACT

This is a final report, for administrative purposes, of studies to be published in the general category of mathematical theory of social groups, with emphasis on the large groups typical of educational communities. Since the study of the large and complicated structures of high school and college communities is difficult by purely empirical methods, they must be represented by mathematical and computer models. Beginning work on such models was the principal activity under the project grant. Abbreviated titles of working papers in the project include: "On the Structure of Large Groups," "The Construction of a Miniature Community," "A Theoretical Framework for the Analysis of Affective Structure," "Sampling Social Structure," "Sociological Applications of Incidence Matrices," "Indifference in Balance Theory," and "Youth Culture." Publication plans for the final versions of these and other working papers in the project will be available by winter, 1972-73, from the Institute of Behavioral Science, University of Colorado. (BW/Author)

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Final Report

Project No. O-0444  
Grant No. OEG-8-70-0203

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STUDIES IN THE THEORY OF LARGE GROUPS

June, 1972

U.S. DEPARTMENT OF HEALTH, EDUCATION, AND WELFARE

Office of Education

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**Final Report**

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**STUDIES IN THE THEORY OF LARGE GROUPS**

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Boulder, Colorado 80302

June, 1972

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## ABSTRACT

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## STUDIES IN THE THEORY OF LARGE GROUPS

This project consisted of theoretical studies, mostly mathematical, of problems in understanding large social groups such as high school and college communities. In a high school of 2,000 students, for example, up to 4,000,000 relationships between different pairs of individuals are possible. In a university of 20,000 students, the corresponding figure is 400,000,000 possible relationships in the campus community. Since these peer relationships are among the most important influences in education, and among the most unpredictable, it is disconcerting that very little is understood about their patterns--how they are formed, what kind of structures emerge, how these serve to integrate some students and isolate others, and so on. This is due, in part, to the difficulty of studying such large and complicated structures by purely empirical (observational) methods. In the nature of a problem of this size, it must be represented by mathematical and computer models.

Beginning work on such models was the principal activity under the project grant. It was done by members of the faculty of mathematical sociology at the University of Colorado (John Bramsen, Robert C. Hanson, Thomas F. Mayer, and William N. McPhee) and two of their colleagues (Patrick Jobes, of the University of Colorado, and Norris Larson, of Loyola University). Since the work was done by individuals, involving technical problems best known by each, a report by each of them on his activities under the grant follows below (in order by expenditures under this grant).

Much of the work is unfinished, and all of it needs revision; so publication plans will not be decided until the end of 1972 or early 1973. Information may be obtained by writing William N. McPhee, Institute of Behavioral Science, University of Colorado, Boulder, Colorado, 80302.

### Report on individual activities: W.N. McPhee

All of my work under this grant will be included in a long paper (or short monograph) entitled "On the Structure of Large Groups," with Barbara S. Dunn, graduate research assistant on this project. It is an expository review and extension of the principal mathematical methods used by sociologists on such problems.

The paper is organized around the analysis, by each method in turn, of an empirical case study made especially for this purpose. It was a case study of all the daily interactions among people in a small office building on campus--about a third of them students, a third staff, and a third faculty--over forty days at intervals

in the growth of friendships among people working together in this (new) group. The task of such a study, then, is to explain the social structure that finally emerges.

Methods illustrated include the logic of relations, Boolean algebra, graph theory, matrix theory, and stochastic difference equations. The latter, for instance, attempt to reproduce the structure (probabilities) of interaction in the matured group from a starting point of no interaction at all, i.e. among previously unrelated individuals.

A first draft of the manuscript will be available by the end of summer, 1972.

Report on individual activities: Thomas F. Mayer

My main efforts under this grant have focussed on (a) the analysis of affective structures, and (b) the preparation of a literature review and theoretical synthesis pertaining to mathematical models of social structure.

An affective structure is, quite simply, a network of liking and disliking among members of a group. My colleague, Norris Larson, and I were concerned with how affective structures came into existence, how they changed, what constituted stable and unstable affective structures, and methods for determining a structure's degree of stability.

We take as our starting point the concepts of balance and clusterability and attempt to generalize these notions to encompass relationships intermediate between strict liking and strict disliking, as well as multi-dimensional relationships. My particular emphasis has been the characterization of an affective structure by means of the eigenvalues of the affective structure matrix. I have been able to establish a number of interesting results including a relationship between the eigenvalues and the cycle system of the affective structure. I have conjectured, but have not been able to prove, several further propositions. As a result of this research, I think it likely that the most important structural and dynamic properties of affective systems can be simply described using eigenvalues.

Professor Larson and I have produced two papers on the analysis of affective structures, "A Theoretical Framework for the Analysis of Affective Structures: I" and ". . .II;" two other papers on this topic are still in progress. Completion of these papers must await solutions to certain problems which thus far have eluded us.

My second major endeavor under the aegis of the Office of Education grant is the preparation of a manuscript on mathematical models of social structure. The literature on structural models is highly diverse including material on clique formation, relationship networks, social stability, organizational structure, power relationships, and kinship systems. I have attempted to review this literature and extract the principle theoretical results. Virtually all of these results pertain to the manner in which large groups organize themselves under various circumstances. When completed, the manuscript will be published by Bobbs-Merrill.

Finally, I should mention the considerable energy which was devoted to an information theoretic analysis of communication processes of large groups. This effort to an information theoretic approach to causal analysis, but it did not achieve anything substantial with respect to the analysis of large groups. It is possible that the information theoretic approach can be fruitful, but as of the present moment I have not been able to make it work.

Report on individual activities: John Bramsen

The work in which I have been engaged has resulted in two papers, both of which are concerned with the theory of structural balance as applied to the study of group structure; each paper is still incomplete.

The first paper, which was delivered at the 1971 meeting of the Rocky Mountain Social Science Association, is titled "The Importance of Indifference." The principal contribution of this paper, I believe, is a suggestion for a new and better method of testing. The method is applied to some sample data in order to provide an example, but I believe that a lot more data of an appropriate form would be required for a full-fledged test.

The other paper is titled "Two Sociological Applications of Incidence Matrices." Although it has been reproduced for the purpose of getting criticism from my colleagues, the last section of the paper has not yet been finished. This paper is entirely theoretical in content. It contains some mathematical results of interest, but the primary contribution, I think, is to suggest the use of a representation of social structure which has been passed by in the social sciences and which has virtues lacking in other mathematical representations. The last part of the paper will be concerned with the use of this representation in the development



of a computer algorithm to find a theoretically smallest set of changes in a structure which when made will result in balance.

Report on individual activities: Norris Larson

Under support of the Office of Education grant, in part, I collaborated with Thomas F. Mayer in working on theoretical problems connected with applying balance theory to the analysis of social groups. Over the period 1970-71 we wrote two papers under joint authorship, "A Theoretical Framework for the Analysis of Affective Structures: I" and ". . .II." I am still working with Mayer on technical problems raised in these general papers. For example, I have derived a characterization of decomposable sociometric structure matrices in terms of their eigenvalues.

I have also developed two other working papers. One is a short note titled "On the Concept of The Structure of a Group." The core of the paper is a multi-dimensional construction of the concept of the total social structure of a group.

The second and longer working paper, "Some Notes on the Problems of Sampling Social Structures," was prepared during 1971. The paper develops a framework for classifying various approaches to sampling structures. Using this typology as a starting point, the paper then outlines three new approaches for obtaining information about the structure of a group which do not require interviewing every member of the group. I am now applying some of the consulting funds made available to me under the grant to conduct an empirical investigation into the usefulness of one of these three approaches. The collection of these data is under way and a write-up should be complete by fall, 1972.

Report on individual activities: Patrick Jobs

My participation in the Office of Education project, "The Theory of Large Groups," was largely spent in three tasks. First, considerable time was spent in the development of a general working bibliography relating to youth culture. Second, a manuscript attempting to relate specific developments and issues specific to the youth culture was written. Finally, time and effort expended on the two items referred to above led me to begin revision of earlier empirical work concerning youth culture, education, and socio-economic status. Analysis of this material is nearly complete and submission for publication should follow this summer.



Report on individual activities: Robert C. Hanson

In the process of building a computer simulation model whose purpose was to reproduce the adjustment processes of rural migrants who have moved to a large city, two procedural problems had to be resolved: (1) how could data from a small sample of migrants be utilized to create a realistic representation of the larger community of peers who interacted with them as friends and associates, and (2) how could data from the sample of migrants be used to construct realistic representations of the physical and economic environment in which they now lived?

My working paper on the problem is entitled, "The Construction of a Miniature Ethnic Community and its Urban Environment;" the paper illustrates, by means of a case history example, how a miniature ethnic community was created, and how job history data were used to construct realistic "world of work" features of the urban environment. These examples show how characteristic features of a large social system may be built up from a small sample of relevant information.